Precalculus (Math 097, 107 or 115) Readiness Check

The intent of these exercises is to help you decide whether you are ready for the upgrading course MATH 097, the university transferable precalculus courses (MATH 107 or MATH 115) or whether you should first upgrade your math skills by refreshing with MATH 077 Algebra & Triangle Trigonometry.

All the questions in the following exercises have full solutions. If you find yourself frequently turning to the solutions to help you answer the questions, this is a sign that your background is deficient on this topic. It is important that you be honest with yourself; that is, are you just a bit rusty and the material will come back to you or do you need a comprehensive review. It is unrealistic to think that you can relearn algebra and trigonometry at the same time as you are learning the more advanced concepts and methods in a precalculus course.

¶ If you struggle with most of the problems or do not remember a large proportion of this material, then you should consider registering in MATH 077 to upgrade your level of mathematical skill.

¶ If you can work through many of the questions but it takes a while and you struggle with some of them, then likely you are ready for either MATH 097 or MATH 107. These are the easier of the precalculus courses and MATH 107 is designed to prepare you for MATH 108 (Applied Calculus).

¶

E.1

1. $\frac{5}{2}x - 7 = 18$

2. $\frac{7}{5}(x-1) - \frac{3}{2}(x-2) = 1$

3. $6w - 5d + 7h = 80$ for w

4. $a = \frac{3}{b}(b - y)$ for "b"

E.2

1. $2x^2 = 14x$

2. $3x^2 = 5x - 1$

3. $3\ ^{2}\ 4\ \ 1\ \ 0$, by completing the square.

E.3

1. $8 - 3x$ ¢ 2

2. $-\frac{1}{2}$ ¢ $\frac{3-2x}{2} < \frac{5}{4}$

E.4

1. $\frac{8}{y} - \frac{1}{3} = \frac{5}{y}$

2. $\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{x^2 - 6x + 8}$

E.5

1. $\sqrt{2x+5} = 7$

2.

F.1

1. Passes through the points $(-2, 3)$ and $(5, -6)$. Write your answer in standard form.

2. Perpendicular to line whose equation is $y = \frac{3}{4}x - 5$, and contains the point (-3, 1).

3. Parallel to the line whose equation is $3x + 5y = 12$, and passes through the point [illegible]

H.2

1. $\sin 24.5^0$

2. $\sec\left(121.7^0\right)$

H.3 Solving Right Triangles:

If the hypotenuse of a right triangle has a length of 15 and one of the angles is 27 degrees then find the lengths of the remaining two sides and the size of the measure of the other angle.

SOLUTIONS

A.1

1. $$-\{2-[4-(3-5)]-(5-2)\} = -\{2-[6]-(3)\}$$
$$= -\{-7\}$$
$$= \boxed{7}$$

2. $$-2^4+(-2)^3-2^{-2} = -16-8-\frac{1}{2^2}$$
$$= -24-\frac{1}{4}$$
$$= \frac{-96}{4}-\frac{1}{4}$$
$$= \boxed{\frac{-97}{4} \quad \text{or} \quad -24\,{}^1\!/_4}$$

3. $$\frac{1}{3^{-1}-4^{-1}} = \frac{1}{\frac{1}{3}-\frac{1}{4}} = \frac{1}{\frac{1}{12}} = \boxed{12}$$

A.2

1. $$a^4-2bc = (-2)^4-2(3)(-4)$$
$$= 16+24$$
$$= \boxed{40}$$

2.

B.1

1. $$\left(-3x^5y^{-2}z^0\right)\left(8x^3y\right)$$
$$= -24x^8y^{-1}$$
$$= \boxed{\frac{-24x^8}{y}}$$
(Recall $z^0 = 1$)

2. $$\left(\frac{a^2b^{-3}c}{2a^{-1}b^{-2}c^3}\right)^{-4} = \left(\frac{a^3}{2^1b^1c^2}\right)^{-4}$$
$$= \frac{a^{-12}}{2^{-4}b^{-4}c^{-8}} = \boxed{\frac{16b^4c^8}{a^{12}}}$$

B.2

1. $$\frac{x^{1/3}x^{-5/3}}{x^{-2/3}} = x^{1/3-5/3-(-2/3)}$$
$$= x^{-2/3}$$
$$= \boxed{\frac{1}{x^{2/3}} = \frac{1}{\sqrt[3]{x^2}}}$$

2. $$\left(x^{1/3}y^{3/2}\right)\left(x^{-1/2}y^{1/4}\right)^2 = \left(x^{1/3}y^{3/2}\right)\left(x^{-1}y^{1/2}\right)$$
$$= x^{\frac{1}{3}-\frac{3}{3}}y^{\frac{3}{2}+\frac{1}{2}}$$
$$= x^{-2/3}y^{4/2}$$
$$= \boxed{\frac{y^2}{x^{2/3}} = \frac{y^2}{\sqrt[3]{x^2}}}$$

C.1

1.

2. $-\sqrt{75x^3y^7} = -\sqrt{25\cdot 3\cdot x^2\cdot x\cdot y^6\cdot y}$

$= \boxed{-5xy^3\sqrt{3xy}}$

3.

C.2

1.

2. $\left(\frac{\sqrt{5}-\sqrt{7}}{2\sqrt{5}+\sqrt{7}}\right)\left(\frac{2\sqrt{5}-\sqrt{7}}{2\sqrt{5}-\sqrt{7}}\right) = \frac{10-\sqrt{35}-2\sqrt{35}+7}{20-7}$

$= \boxed{\frac{17-3\sqrt{35}}{13}}$

D.1

1. $3\left[3-2(x-2)-2(3-4x)\right]$

$= 3\left[3-2x+4-6+8x\right]$

$= 3\left[1+6x\right]$

$= \boxed{18x+3}$

2. $-4(2x-3)^2-(5x-1)(2x+3)$

$= -4(4x^2-12x+9)-(10x^2+13x-3)$

$= -16x^2+48x-36-10x^2-13x+3$

$= \boxed{-26x^2+35x-33}$

3. $(3x-4)(7x^2+5x-1) = 21x^3+15x^2-3x-28x^2-20x+4$

$= \boxed{21x^3-13x^2-23x+4}$

D.2

1. ^{2} $\boxed{(\quad)(\quad)}$

2. $x^2-7x+12 = (x-3)(x-4)$

3. $6x^2+26x-20 = 2(3x^2+13x-10)$

$= 2\left(\frac{3x+15}{3}\right)\left(\frac{3x-2}{1}\right)$

$= \boxed{2(x+5)(3x-2)}$

4. $x^2y^2+ab-ay^2-bx^2$

$= x^2y^2-ay^2-bx^2+ab$

$= y^2(x^2-a)-b(x^2-a)$

$= \boxed{(x^2-a)(y^2-b)}$

D.3

1. $\frac{2x^2-8}{x^2-4x+4} = \frac{2(x^2-4)}{(x-2)^2}$

$= \frac{2(x-2)(x+2)}{(x-2)^2}$

$= \boxed{\frac{2(x+2)}{x-2}},\ x \neq \pm 2$

2. $\frac{x^2-5x-6}{x^2-6x}\cdot\frac{6}{12x+12} = \frac{(x-6)(x+1)}{x(x-6)}\cdot\frac{6}{12(x+1)}$

$= \boxed{\frac{1}{2x}}$

3.

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D.4

1.

$$\frac{21x^3 - 35x^2 + 14x - 7}{7x}$$
$$= \frac{21x^3}{7x} - \frac{35x^2}{7x} + \frac{14x}{7x} - \frac{7}{7x}$$
$$= \boxed{3x^2 - 5x + 2 - \frac{1}{x}}$$

2.

$$\begin{array}{r} x^2 - 2x + 4 \\ 3x + 1\overline{)3x^3 - 5x^2 + 10x - 3} \\ -\underline{\left(3x^3 + x^2\right)} \\ -6x^2 + 10x \\ -\underline{\left(-6x^2 - 2x\right)} \\ 12x - 3 \\ -\underline{\left(12x + 4\right)} \\ -7 \end{array}$$

$$Q(x) = x^2 - 2x + 4$$
$$R(x) = -7$$

E.1

1.

$$\frac{5}{2}x - 7 = 18$$
$$5x - 14 = 36$$
$$5x = 50$$
$$\boxed{x = 10}$$

2.

3.

$$6w - 5d + 7h = 80 \quad \text{for w}$$
$$6w = 5d - 7h + 80$$
$$w = \boxed{\frac{5d - 7h + 80}{6}}$$

4.

$$a = \frac{3}{b}(b - y) \text{ for "}b\text{"}$$
$$ab = 3b - 3y$$
$$3y = 3b - ab$$
$$3y = b(3 - a)$$
$$\boxed{b = \frac{3y}{3 - a}}$$

E.5

1.

2.

$$\sqrt{3x+1}-\sqrt{x+4}=1$$
$$(\sqrt{3x+1})^2=(1+\sqrt{x+4})^2$$
$$3x+1=1+2\sqrt{x+4}+x+4$$
$$(2x-4)^2=\left(2\sqrt{x+4}\right)^2$$
$$4x^2-16x+16=4(x+4)$$
$$4x^2-16x+16=4x+16$$
$$4x^2-20x=0$$
$$4x(x-5)=0$$
$$x=0 \quad \text{or} \quad x=5$$
$$\text{Check: } \sqrt{3(0)+1}-\sqrt{(0)+4}\neq 1$$
$$\sqrt{3(5)+1}-\sqrt{(5)+4}=1$$

So, $x=5$, since $x=0$ does not check

F.1

1. $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-6-3}{5-(-2)}=\boxed{-\dfrac{9}{7}}$

using point-slope form:

$y-y_1=m(x-x_1) \Rightarrow y-3=-\dfrac{9}{7}(x+2)$

$7y-21=-9x-18 \Rightarrow \boxed{9x+7y=3}$

2. $m=\dfrac{3}{4}$, since $\perp$, $m=-\dfrac{4}{3}, (-3,1)$

$y=mx+b \Rightarrow 1=\left(\dfrac{4}{3}\right)(-3)+b$

$b=-3, \quad \Rightarrow \quad \therefore \quad \boxed{y=-\dfrac{4}{3}x-3}$

3. $-2x+5y=12$

$5y=2x-12$

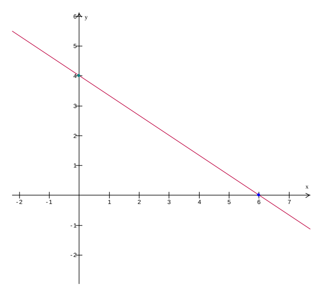
$y=\dfrac{2}{5}x-\dfrac{12}{5} \quad m=\dfrac{2}{5}$, since $//$, $m=\dfrac{2}{5}, (10,-1)$

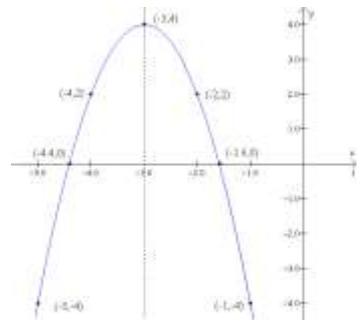
$y=mx+b \Rightarrow -1=\left(\dfrac{2}{5}\right)(10)+b$

$b=-5, \quad \Rightarrow \quad \therefore \quad \boxed{y=\dfrac{2}{5}x-5}$

F.2

1.





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2.

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G

1.

H.1

1. Use Pythagoras to find "a"

$c^2 = 3^2 + \left(\sqrt{7}\right)^2 \Rightarrow c^2 = 16, \therefore c = 4.$

$\boxed{\tan\theta = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}}, \quad \boxed{\csc\theta = \frac{4}{3}}$

H.2

1. $\sin 24.5° = 0.415$

2. $\sec 121.7° = \frac{1}{\cos 121.7°} = -1.903$

H.3

A + B + C =180, so if A is the missing angle then A + 27 + 90 = 180 so A = 63 degrees.

If is the length of the side opposite the $27°$ angle then $\sin 27° = \frac{a}{15} \Rightarrow a = 15\sin 27° = 6.81$

If is the length of the remaining side then $\cos 27° = \frac{b}{15} \Rightarrow b = 15\cos 27° = 13.37$